

The Monty Hall Problem and the “As If” Assumption Of Economics

Yashaswini Shekhawat

St. Stephen’s College

Abstract

The Monty Hall problem has been known to elicit incorrect responses from the majority of people who encounter it. Through this paper, the author formulated the problem as a choice decision, and compared how different cohorts either chose to switch or not switch when offered the choice; with the proportion of females who switched almost equal to proportion of males who switched, while younger people (ages 12-40) chose to switch more often than older people (ages 40 or above). But on the aggregate, people chose to stick with their original choice of door, and only around 7.25% switched, indicating that people do not always make choice decisions that conform to mathematical theory.

Keywords: Monty Hall problem, choice behavior, behavioral economics, “as if” assumption of Economics, optimal decision making, differences between decisions made by people of different genders and age groups.

Acknowledgements

I would like to thank the people without whom this project would not be complete.

First and foremost, I would like to thank Professor Poonam Kalra, head of the economics department, St. Stephen’s College, in whose class I was first introduced to the Monty Hall problem and thought about this idea. She has been a great support and has guided me throughout this project, from its initial conception to its final edit. This project would not have been possible without her.

I would also like to thank all the respondents for taking out their invaluable time to fill out the questionnaire.

Introduction

In 1959, Martin Gardner wrote a column for the Scientific American Journal, which had a brain teaser, presented by Gardner to illustrate how even experts could make blunders in probability theory. The brain teaser given by Gardner in his column is now called the “Prisoners’ Dilemma”, and is touted to be the first instance where a “Monty Hall like” problem was formally presented and published.

In 1963, a television game show premiered called “Let’s make a deal”, in which the host, Monty Hall, played a number of games with contestants, one of which was a game that inspired the “Monty Hall problem” in mathematical literature. In the actual version of the game (and not the mathematical abstract version), contestants would be

presented with three identical doors, any two of which were known to have goats behind them, while one was known to have a car behind it. Contestants were then asked to pick the door that they thought had a car behind it. If the contestants chose a door with a goat, the door was opened immediately, while if the contestant chose a door with the car, Monty opened one of the remaining doors and then asked the contestant if they wanted to switch, often offering a sum of money to facilitate the switch.

The elusive nature of the abstract version of the Monty Hall problem truly hit the mathematical community on September 9, 1990, when Marilyn vos Savant, a columnist for the Parade magazine responded to a question, which is the following version of the game presented in “Let’s make a deal” and this is the version which came to be known as the Monty Hall Problem:

Suppose there are three identical doors in front of you, two of which contain a goat, while one contains a car. You are supposed to pick the door you think has the car behind it. After you pick the door, the host, who knows which door contains what, will open a door he knows has a goat behind it. Now the host gives you a choice; you can either switch over to the other door or you can stick to your initial choice. Should you switch or not?

vos Savant answered that it would be better to switch and by way of explanation, said the following, “Yes, you should switch. The first door has a $1/3$ chance of winning, but the second door has a $2/3$ chance. Here's a good way to visualize what happened. Suppose there are a million doors, and you pick door number 1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door number 777,777. You'd switch to that door pretty fast, wouldn't you?”¹ That answer column caused uproar in not just regular readers of the magazine, but the mathematical community at large. vos Savant received hundreds of letters, stating, sometimes in overly condescending and patronizing tones, that she was completely wrong. It is noteworthy that these replies were not just from the general public, but also from professional mathematicians. Even Paul Erdos, one of the most famous mathematicians of the twentieth century got the problem wrong and also refused to accept the correct answer for quite some time, despite being shown the answer mathematically.²

The reason why so many people consistently gave the incorrect answer, even while understanding the probabilistic nature of the problem, is that they failed to understand the significance of Monty’s revelation. The reasoning adopted by most people was the following: You choose one door; Monty opens a door which has a goat behind it. There remain two doors. Probability that the door you picked has a car behind it is then $1/2$, as there are only two doors remaining, so it doesn’t matter whether you switch or not.

Throughout modeling in economics, a good deal of mathematics, probability and statistics are used, especially in cases where optimal choice decisions are to be made,

¹ Rosenhouse, Jason, *The Monty Hall Problem: The Remarkable story of Math’s most Contentious brain teaser*, (New York, Oxford University Press, 2009), p. 23 and Marilyn vos Savant, *The Power of Logical Thinking*, St. Martin's Press, New York (1996)

² Rosenhouse, *The Monty Hall Problem*, p. 54-55

and the general reasoning behind the usage of standard tools of mathematical optimization in such models is usually the following: People might not know the mathematics that goes into making these decisions, but they act “as if” they knew.

The Monty Hall problem is an instance that shows that people can make suboptimal choices, as against the optimal choices presented by mathematical theory, and thus serves as an important lesson; that we need to be extremely cautious in applying the “as if” notion of rationality in economics and other disciplines that seek to map out choice behavior of humans.

Literature Review

The Monty Hall problem has been approached largely in two ways, one to establish the correct strategy that should be followed to achieve the highest probability of winning a car, while the other approach has been to look into cognitive illusions associated with the Monty Hall problem, and what strategy people employed to win in the Monty Hall problem.

Jason Rosenhouse, in his book *The Monty Hall problem*, explains several ways in which the Monty Hall problem has been analyzed to arrive at the correct strategy. In what he terms the “Classical Monty”, he forms a sample space to be consisting of triples: (1,2,3), where the 1st element represents initial choice, the 2nd element represents the door that Monty opens, while the third element represents the location of the car.³

He states, “Since Monty always opens a door with a goat, we know that the 2nd and 3rd elements of any triple must be different. Likewise, since Monty never opens the door you initially choose, we can assume that the 1st and 2nd numbers are different as well.”

So, we have the following elements in the sample space:

$S = \{ (1,2,1) (1,3,1) (1,2,3) (1,3,2) (2,1,2) (2,3,2) (2,1,3) (2,3,1) (3,1,3) (3,2,3) (3,1,2) (3,2,1) \}$

Now what are the probabilities associated with each of these triples? Are they equally likely?

Consider the elements where you initially choose door number 1.

$P\{(1,2,1)\}$ consists of three events. Now, the probability that you chose door no. 1 is $1/3$ (random choice). Since, the car is behind door no. 1 (probability of which is also $1/3$), then Monty can open either door 2 or door 3. Thus, probability of opening door 2 is $1/2$.

Similarly, we can arrive at all other probabilities in the sample space.

So, the question arises, what is the probability of winning if you switch?

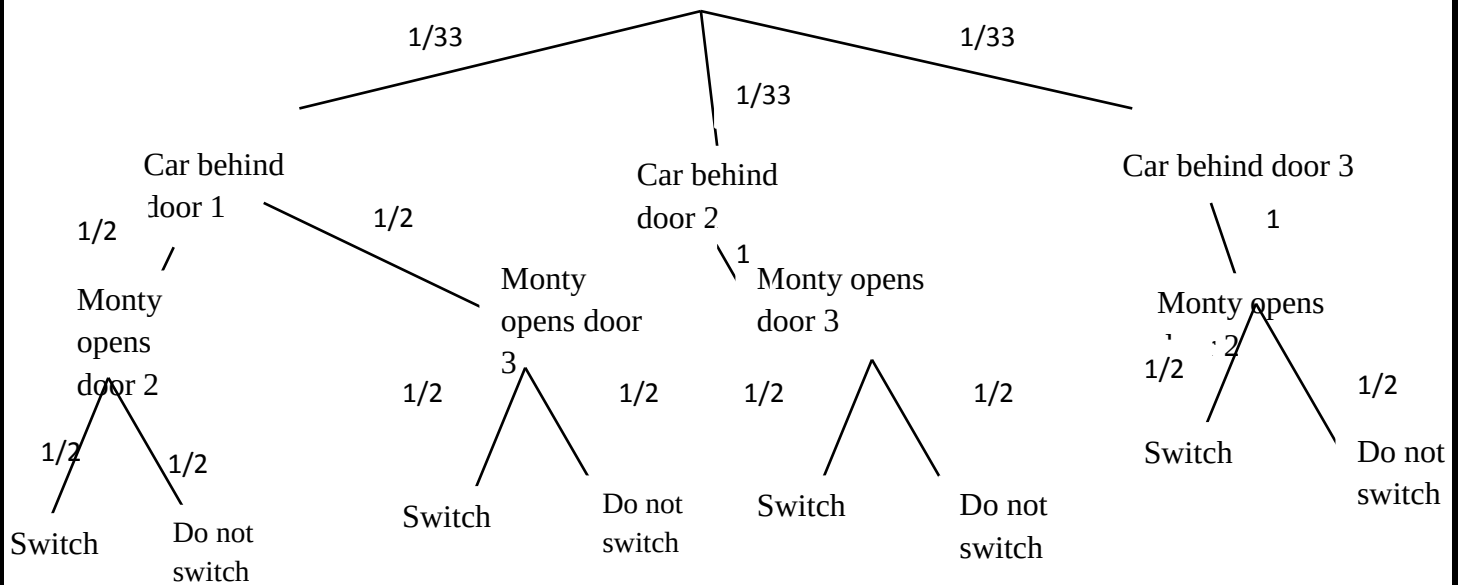
Probability of each of the terms where you can switch and you win is $1/9$. There are 6 such terms. Thus, the probability of switching and winning is $2/3$.

Similarly, the probability of not switching and winning is $1/3$ (as there are 6 terms where you can win by not switching, and probability of each of those terms is $1/18$)

³ Rosenhouse, *The Monty Hall problem*, Chapter 2: Classical Monty

Rosenhouse also uses Bayes theorem to explain why the strategy adopted to win the game should be to switch, in what he terms the “Bayesian Monty”. I will use a tree diagram to briefly explain this method.

Suppose player initially chooses door 1.



Thus, $P(\text{Winning} | \text{You Switch}) = P(\text{Winning} \cap \text{Switching}) / P(\text{Switching})$
 $= (1/6 + 1/6) / (1/2) = 2/3$

Similarly, $P(\text{Winning} | \text{You do not switch}) = 1/3^3$

Bapeswara Rao and Bhaskara Rao take a case of 4 doors, in an attempt to explain the Monty Hall problem, where one conceals a car and the other three conceal goats. You are then told to choose a door at random and then Monty opens a door he knows has a goat hidden behind it. Then, you are given a choice of either switching or sticking to initial choice. After you make your choice, Monty throws open another door which has a goat behind it, and again gives you a choice of either sticking to the door you selected in the previous stage or switching. Rao and Rao claim that in this case, the SLM, or Switching at the last minute strategy should be followed. This is because the probability that you picked a door that has a goat is $3/4$ while probability that you picked a car initially is $1/4$. If you stick with your initial choice, Monty will keep revealing the doors with the goats, and at the last minute you can make a switch to the door that is left.

Mitzenmacher ran a simulation using the software C, where once, he followed the strategy of only switching, which resulted in a win 66.708% of times, while when he ran a simulation following the strategy of not switching, it resulted in a win only 33.326% of times.

³ Rosenhouse, *The Monty Hall Problem*, Chapter 3: Bayesian Monty

Many authors have tried to understand the psychological or cognitive aspect behind the Monty Hall Problem.

Burns and Wieth (2000), tried to understand why the Monty Hall problem creates issues for most people. They theorized that it was because of the causal structure of the Monty Hall problem; that people found it hard to reason out the problem, because it is difficult to grasp intuitively. They used another version of the Monty Hall problem, where there are 3 boxers, and one of them is the best, who will win every time, against any other boxer. You pick the boxer who you think plays the best, and the remaining two fight. You are then given the option of either switching to the winner of that bout, or sticking to your initial choice. They found that 51% of the participants switched in this case, as against 15% switching proportion in the original Monty Hall problem. This shows that people are more likely to solve the problem when the causal structure is easier to understand.

Granberg and Brown selected a sample of 114 undergraduate students and asked them to play the game 50 times and tried to see whether a proper winning strategy was found out by the participants over time. They found out that although there was a learning curve and participants began switching more often, they did not completely transition into the strategy of switching. Thus, they found that suboptimal behavior was persistent; that it reduced over time but did not vanish.

Herbranson and Schroeder made a novel study by analyzing choice decisions of humans and pigeons. For pigeons, there were 100 trials, where pigeons were kept in an operant chamber with three doors. Each door was lit with white light, and location of seed was randomly selected out of the three doors. When a pigeon pecked on a door, then one door was darkened, one not picked by the pigeon. Then, only the initial choice of the pigeon and one other door remained lit. Then, the pigeon pecked at a door again, and accordingly got the seed, if it picked the correct door. According to this study, after sometime, pigeons consistently switched on all trials, whereas humans only switched about $2/3^{\text{rd}}$ of the time.

This is also to do with the gut level feelings of certainty of humans. Watkins, Derks and Dougherty found that participants would feel worse if they switched and lost rather than when they stayed and lost, i.e. regret for actions is greater than regret for inaction in humans, with one participant in their study saying, “It would stink if I switched and found out that I had the right one in the first place.”

Thus, the literature on the Monty Hall problem has shown two things:

1. The correct strategy for winning is undoubtedly switching
2. Humans, when faced with this problem, more often than not, opt for the strategy of not switching.

Most of the research on the Monty Hall problem has been to establish the correct answer to the problem, and of the research on the cognitive and choice aspect of this problem, researchers have not focused on the differences that can arise in choice judgment of different age groups, genders etc. and the research has not been extended to the analyzing optimal choice behavior in economics, which is what this paper seeks to do.

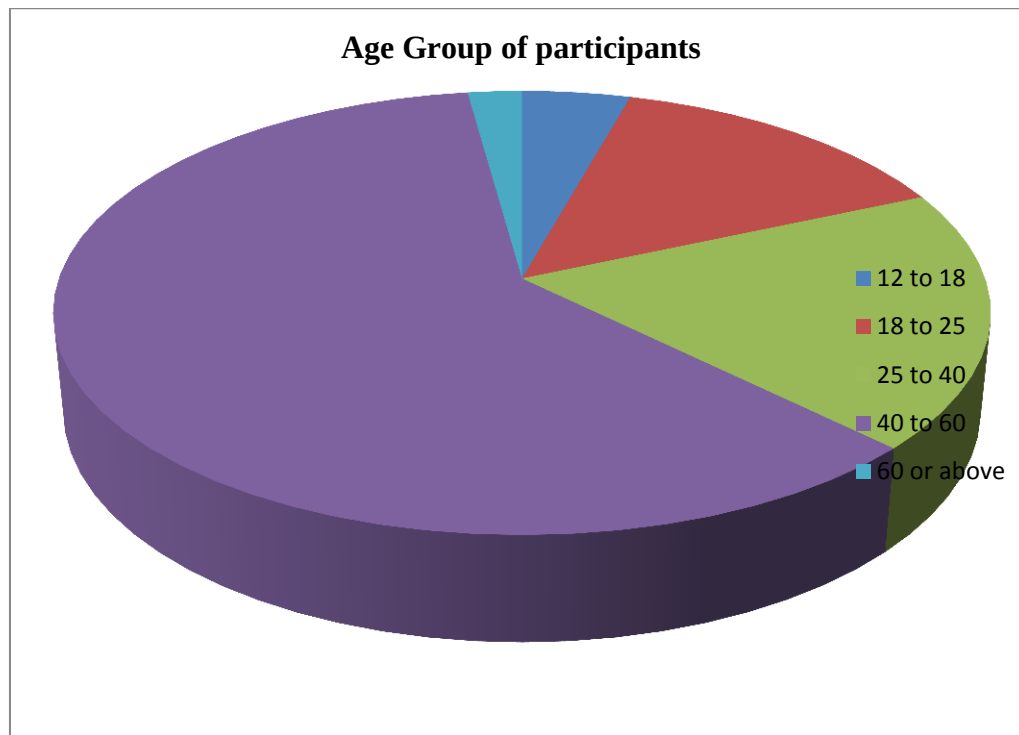
Methodology

Survey

A questionnaire was created based upon a pilot survey of around 30 individuals. The questionnaire was created as a Google form and sent out to many individuals, who voluntarily participated in the study. A total of 276 responses were recorded and then analyzed. (Refer to Appendix A)

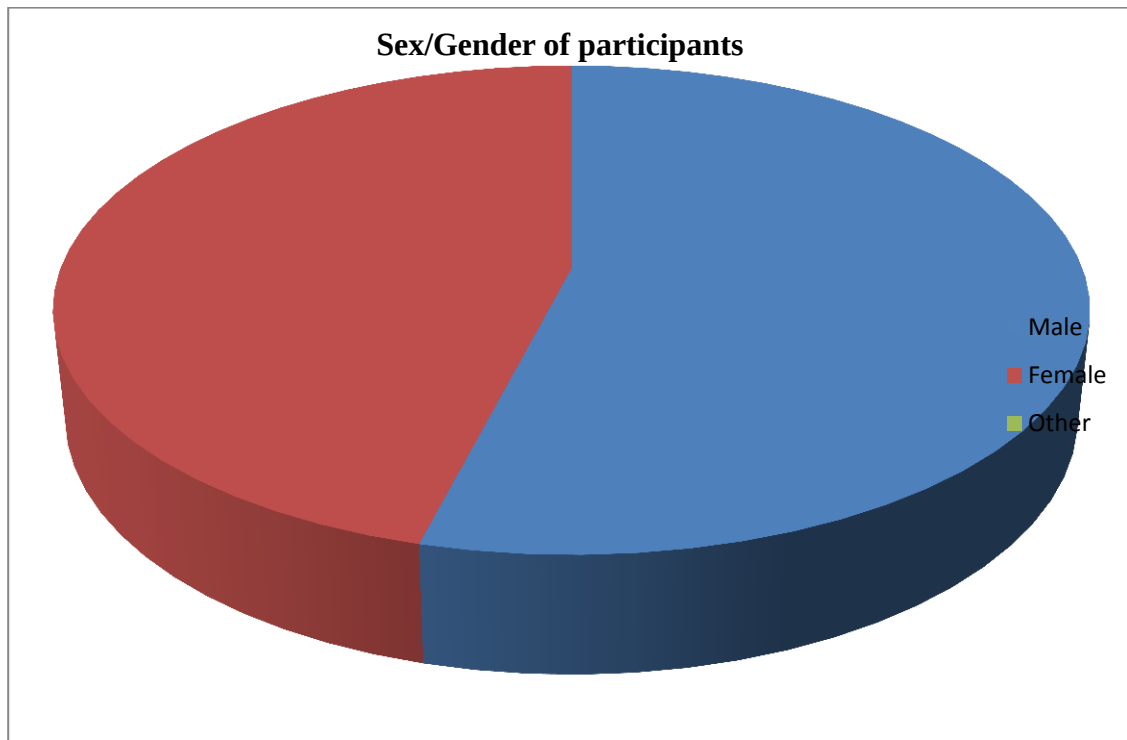
The sample was a snowball sample, consisting of randomly selected persons of various age groups, who each then contacted others to participate in the study.

Age Group of Participants



- 12-18: 12 out of 276
- 18-25: 39 out of 276
- 25-40: 52 out of 276
- 40-60: 167 out of 276
- 60 or above: 6 out of 276

Sex/Gender of Participants



- Males: 149
- Females: 127
- Others: 0

Variables

Apart from analyzing the choice of switching and not switching in general, the following variables have been considered and their impact on the above choice has been examined.

1. *Gender*: Gender is often thought to affect the way different people take decisions and approach choice problems. The popular understanding still remains that males take decisions and think about problems in a more logical and mechanical manner when compared to females who tend to rely more on their instincts. The author wished to test whether any such relationship appeared in this particular choice problem and whether male responses were significantly different from female responses.

The author believes that this particular perception of difference between male and female understanding is purely a societal bias, and therefore expected to see no significant difference between male and female responses.

2. *Age*: Age plays a crucial role in the choice behavior of humans. People belonging to different age groups have a different understanding of this world, and form judgments and perceptions differently. This difference is often also reflected in the way people take decisions. In this particular problem, the author wished to see whether younger people (ages 12-40) approached the Monty Hall problem differently than older people (ages 40 onwards) and which people belonging to

which age groups could determine the answer correctly more often than others, if such a difference existed.

Simulation

An R simulation was also run, to corroborate the correct strategy in this game, with 276 trials run twice, to exactly match the number of responses in the survey.

Initially, the strategy followed was to switch on all 276 trials and observe how many resulted in a win, given that the car and goats were randomly allotted.

Then, the strategy was changed to sticking with the original door in all 276 trials and the number of times this strategy resulted in a win was also recorded.

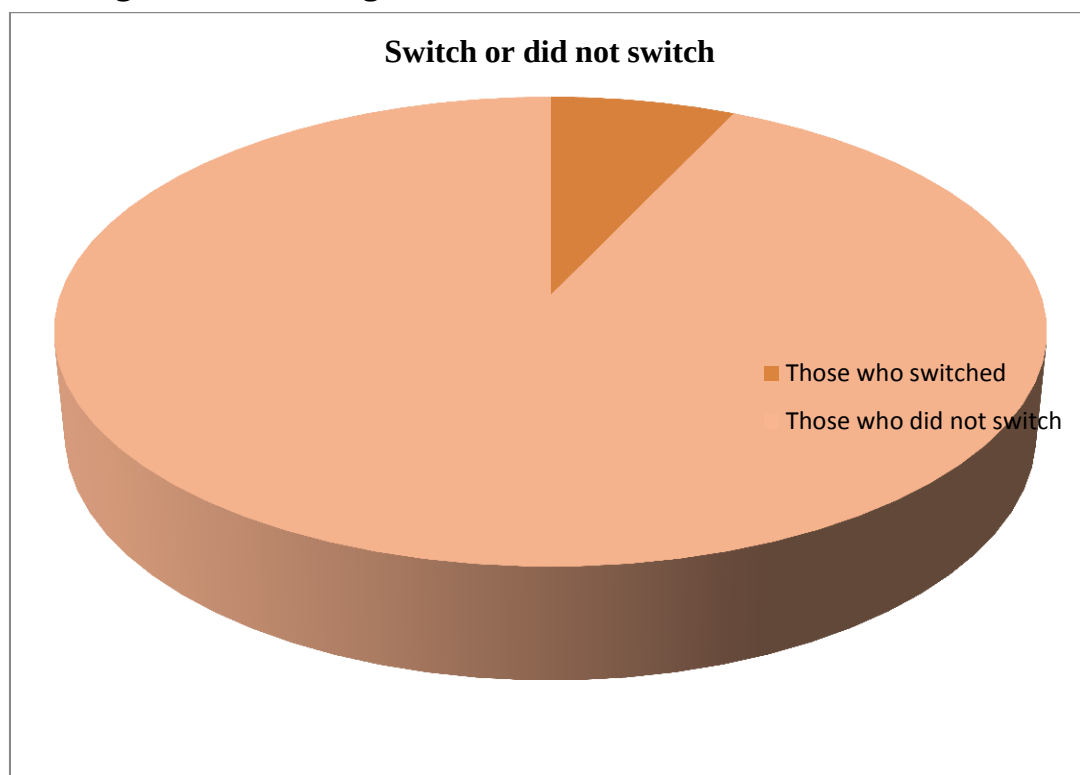
Findings and Analysis

Survey Results

1. Number of people who picked doors A, B and C

As is always taken, number of people who picked each door was approximately $\frac{1}{3}$ rd of the total sample, with approximately 35% picking door A, 33% picking door B and 32% picking door C.

2. Switching or not switching



- Those who switched: 20
- Those who did not switch: 256

The number of people who switch doors when given the option is just 20 out of 276, which is around 7.25%. Thus, the probability of people switching is not $\frac{1}{2}$, but much lesser.

Around 90% of the people who chose to stick with their original choice said that gut feeling guided their decision; either they used these words directly, or rephrased it by saying that they felt that their initial choice was correct. Around 7% said that there were equal chances of getting either car or goat after one gate had been eliminated, so why switch, while the rest gave no specific reason.

What is apparent from this is that this problem in particular, is not viewed by participants as a problem of probability, and since they cannot connect the additional information in any manner to the problem at hand, they tend to stick with instinct and gut feeling.

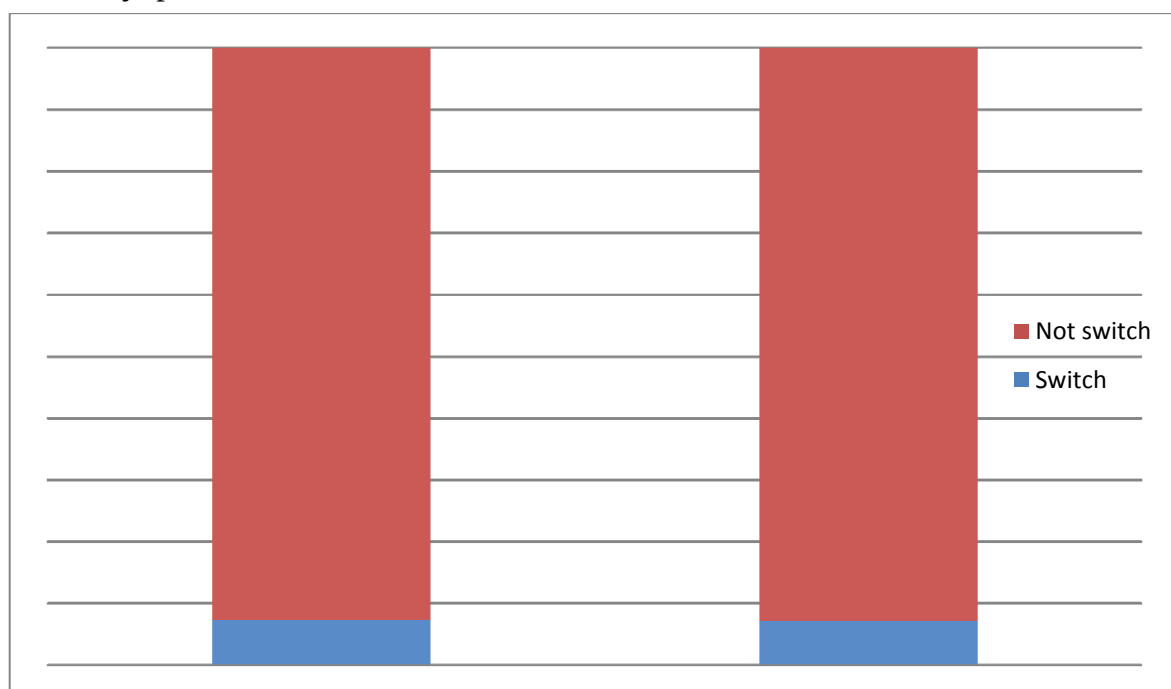
Of the 20 people who switched to the other door, 11 knew the game and how it worked and so answered accordingly. The rest of the participants either said they had a hunch that the other door might have the car behind it, while the rest provided no specific answer. Only one participant worked out the probabilities correctly, saying that he had a chance of $\frac{2}{3}$ of getting the door wrong and hence should switch.

3. Male vs. Female

Often it is touted that males “approach” problems in a more logical manner than females, who rely more on their instincts. This is not the case in this mathematical problem. Both males and females have almost equal difficulty in understanding the structure of the problem, and both males and females said that they chose to stick to their choice because of gut feeling.

Out of 127 females, 9 chose to switch, i.e. 7.1% chose to switch, while out of 149 males, 11 chose to switch, i.e. 7.3% chose to switch.

Thus, in this problem also we see that males and females both approach the problem similarly, provided similar rationale for their choices, and tended to stick to instinct.



4. Age Groups

- a) Of 11 participants in the 12-18 years of age category, 2 decided to switch (16.66%); one gave the reason of guessing, while the other was the only participant who worked out the probability of the problem correctly and gave that as a justification for switching (9.09%).
- b) Of 39 participants in the 18-25 years of age category, 5 decided to switch (12.82%), of whom 3 knew how the game worked (7.69%), and others had a hunch that the other door might be correct.
- c) Of 52 participants in the 25-40 years of age category, 4 decided to switch (7.69%), of whom 2 knew how the game worked (3.84%), while the rest either did not provide a specific answer or said that they had a hunch.
- d) Of 167 participants in the 40-60 years of age category, 9 decided to switch (5.38%), of whom 6 knew how the game worked (3.6%), while the others did not provide specific answers.
- e) Of 6 participants in the 60 or above category, none switched.

Age Groups	Number of participants	Those who switched	Those who did not switch
12-18 years	11	2 (16.66%)	9 (83.34%)
18-25 years	39	5 (12.82%)	34 (87.18%)
25-40 years	52	4 (7.69%)	48 (92.31%)
40-60 years	167	9 (5.38%)	158 (94.62%)
60 or above	6	0 (0%)	6 (100%)

There is a clear indication that the younger participants switched more often than the older ones, although it can be said that more of the younger participants had knowledge of how the game worked than the older participants (in percentage terms). This could also be because a lot of the younger participants who were targeted were school and college going students who may have recently encountered this problem in their probability classes.

Even then, across age groups too, the probability of switching is never higher than around 16%, which shows that across age groups people tend to rely more on their instincts, although younger participants can be claimed to have the advantage of recent probability and statistics classes to help them in this regard.

Thus, in general, across genders and age groups, participants showed a high tendency to rely on their own subjective judgment and view of the problem. Thus, it can be conclusively stated that the notion that people will always intuitively act in a manner that corresponds with mathematical results is not true, as mathematical results are often arrived at by rather counterintuitive measures.

Computer Generated Simulation Results

Using Simulation in R, the following results were found:

1. By running 276 trials and following the strategy of not switching for all, won the game 32% of times and lost 68% of times
2. By running 276 trials and following the strategy of switching for all trials, won the game 64% of times and lost 36% of times.

Thus, even in this case where sample size is 276, we establish that if all the people had followed the strategy of switching, around 64% of them would have won, and around 36% would have lost as against the strategy of not switching, which leads to loss more times than victory.

Other Findings

1. Confidence Interval for Population Proportion:

X = number of people who switch

$n = 276$

p = actual proportion of those who switch

Then, $\hat{p} = X/n = 0.07246$

Since, X is binomial and n is large, then, $X \sim N(np, npq)$

Then, $\hat{p} \sim N(\hat{p}, \sqrt{pq/n})$

Thus, using a 95% traditional confidence interval, we have,

$$\begin{aligned} p \pm z\sqrt{pq/n} &= 0.07246 \pm 1.96\sqrt{[(0.07246)(0.92754)/276]} \\ &= 0.07246 \pm 1.96\sqrt{(0.000243512857)} \\ &= 0.07246 \pm 0.03059 \end{aligned}$$

Thus, Confidence interval is (0.04187, 0.10305)

2. Confidence Interval for Male and Female Population Proportion:

X_1 = number of males who switch

X_2 = number of females who switch

$n_1 = 149$

$n_2 = 127$

where n is the sample size of males and females respectively.

p_1 = true proportion of males who switch

p_2 = true proportion of females who switch

$\theta = p_1 - p_2 \sim N[p_1 - p_2, \sqrt{(p_1q_1/n_1 + p_2q_2/n_2)}]$

Thus, a 95% confidence interval is obtained using the traditional method for θ , (which is the difference between the true proportion of males and females who switch) which is, (-0.05915, 0.06315).

Thus, it can be said with confidence level of 95% that 0 will belong to the interval of the true difference between population proportions of males and females who switch. Thus, the true proportion of females and males who switch will be almost equal.

Conclusion

The objective of this paper was established at the very beginning as being twofold:

- a) Examining the choice of switching and not switching of people in the Monty Hall problem and establishing differences across genders and age groups, if any existed.
- b) Placing the obtained result in the context of optimal and suboptimal choices made by individuals as against optimal and suboptimal results according to mathematics.

This study showed that people made the choice of sticking to their initial choice across genders and age groups, and although a relatively larger proportion of younger participants opted to switch, this can be attributed to the fact that the targeted population of younger participants was the school and college going students.

Of the 276 participants, only 7.25% chose to switch, and of the 256 participants who did not switch, around 90% said that they did not do so because of gut feeling or instinct.

According to mathematical theory, there exists a clearly established optimal strategy, as discussed in the literature review, which is to switch at all times. Participants however did not switch most of the time, sticking with their instinctive choice. Most did not even approach the problem as a probability problem. This clearly shows that we cannot always assume that intuition will guide individuals in making mathematically optimal choices.

Thus, when examining human choice behavior, we need to be extremely careful before working with the assumption that the optimal strategy established by mathematics is the one that people will also follow, as they will act “as if” they knew the mathematics, because mathematics can often be counterintuitive.

This is especially true of economics, where in both macroeconomics and microeconomics, the “as if” assumption of rationality is prevalent i.e., people are rational if they take decisions according to certain mathematically optimal strategies, else they are not. This is also something we need to be wary of and we should define rationality in accordance with human behavior and not mathematical optimization.

Appendix A

Questionnaire

1. What is your age?
 - a) 12-18
 - b) 18-25
 - c) 25-40
 - d) 40-60
 - e) 60 or above
2. What gender do you identify with?
 - a) Male
 - b) Female
 - c) Other:
3. You are asked to play a game. There are three doors in front of you labeled A, B and C. Two have them have a goat behind them, while one has a car behind it. You do not know which door has what behind it and you are asked to pick the door you think has the car behind it. Which door do you pick?
 - a) Door A
 - b) Door B
 - c) Door C
4. Now that you have picked Door A/B/C (according to what the participant picked), it is revealed that Door B/C (according to what the participant picked; door A had the car behind it) has a goat behind it. You are now given an option: you can either choose to stick to your initial choice or you can switch over to the other door. What do you do and why?
 - a) Stick to initial choice
 - b) Switch over
 - c) Reason:

Note: All questions and subparts were compulsory.

References

1. Rosenhouse, Jason. *The Monty Hall Problem: The Remarkable Story of Math's Most Contentious Brain teaser*. Oxford University Press, 2009.
2. V.V. Bapeswara Rao and M. Bhaskara Rao, A three door game show and some of its variants, *The Mathematical Scientist*, Vol. 17, no. 2 (1992), p. 89-94
3. Mitzenmacher, Michael, The Monty Hall Problem: A Study, *Research Science Institute* (1986)
4. Burns, Bruce & Weith Mareike, The Collider Principle in Causal Reasoning: Why the Monty Hall problem is so hard, *Journal of Experimental Psychology General*, Vol. 133, no. 3 (2004), p. 434-449

5. Granberg, Donald, & Brown, Thad, The Monty Hall Dilemma. *Personality and Social Psychology Bulletin*, Vol. 21, (1995) p. 711-723
6. Herbranson, W. T., & Schroeder, J., Are birds smarter than mathematicians? Pigeons (*Columba livia*) perform optimally on a version of the Monty Hall Dilemma. *Journal of Comparative Psychology*, Vol. 124, (2010) p. 1-13
7. Watkins, Ana, Derks, Peter & Dougherty, Michael, Reasoning in the Monty Hall Problem: Examining Choice Behavior and probability judgments. *Thinking and Reasoning*, Vol. 9, no. 1, (2003), p. 67-90
8. Gardner, Martin, "Mathematical Games" column, *Scientific American* (1959)
9. Marilyn vos Savant , "Ask Marilyn" column, *Parade Magazine* (1990)